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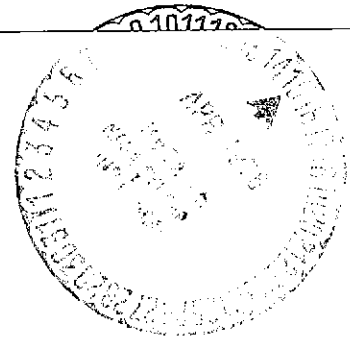*Lockheed*

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USER'S GUIDE AND DESCRIPTION
OF THE STREAMLINE DIVERGENCE
COMPUTER PROGRAM

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Contract NAS9-13429

by

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FOREWORD

This document presents the results of work performed by personnel of the Fluid Mechanics Section of the Lockheed-Huntsville Research & Engineering Center, Huntsville, Alabama. This work was performed for the Aerodynamic Systems Analysis Section of the NASA-Johnson Space Center, Houston, Texas, (Contract NAS9-13429) in support of Space Shuttle plume impingement studies. The technical monitor for this contract is Mr. Barney B. Roberts, EX-32.

SUMMARY

The Streamline Divergence Program was developed to demonstrate the capability to trace inviscid surface streamlines and to calculate "outflow corrected" laminar and turbulent convective heating rates on surfaces subjected to exhaust plume impingement. The analytical techniques used in formulating this program are discussed. A brief description of the Streamline Divergence program is given along with a user's guide. The program input and output for a sample case are also presented.

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Section 1
INTRODUCTION

Most current theories used in predicting convective heat transfer rates on body surfaces subjected to exhaust plume impingement do not utilize an adequate outflow correction theory. When a plume impinges on a surface at some angle of inclination, there is an outflow-induced thinning of the boundary layer, and a corresponding increase in the convective heating rates to the surface. Application of current methods for predicting convective heat transfer (without an adequate outflow correction theory) often results in the predicted heating rates being considerably lower than the experimentally measured heating rates. Without an outflow analysis to determine the severity of the thinning of the boundary layer and the corresponding increase in convective heating rates it is very difficult to make experimental-theoretical comparisons. Under such circumstances, experimental-theoretical comparisons can result in erroneous conclusions. For example, convective heating rates are determined by both turbulent and laminar theory. The convective heating rates determined by turbulent theory are found to compare more favorably with experimental data than by laminar theory. The conclusion is then made that the experimental data are turbulent which may actually be laminar. When proper outflow correction theory is applied, such erroneous conclusions can hopefully be avoided. The results determined by laminar theory and by turbulent theory should compare favorably with the laminar and turbulent data, respectively.

The computer program described in this report has been developed to: (1) demonstrate the capability to trace inviscid surface streamlines and calculate "outflow corrected" laminar and turbulent heating rates on a body subjected to rocket exhaust plume impingement (It is capable of analyzing impingement due to any supersonic flow field); and (2) improve the existing

convective heat transfer prediction techniques (which do not apply outflow correction theory) of the Lockheed-Huntsville Plume Impingement Computer Program (PLIMP) (Ref. 1-1).

The analytical methods used in performing the inviscid surface streamline and convective heat transfer calculations are described in Section 5 of Ref. 1-2. The Streamline Divergence program can be used to make: (1) comparative evaluations of the effects of plume impingement due to different propulsion system configurations; (2) parametric studies; and (3) vehicle design studies.

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Section 2
PLUME IMPINGEMENT ANALYSIS USING THE STREAMLINE DIVERGENCE METHOD

An analysis to trace inviscid surface streamlines and calculate "out-flow corrected" heating rates on surfaces subjected to exhaust plume impingement is a rather detailed procedure. The Streamline Divergence program is the means by which the final results are obtained. This section is intended to familiarize the program user with the data used by the Streamline Divergence program, and the means by which the data are generated. A brief computational scheme is presented in the following paragraphs.

The prominent features of the computational scheme required in order to completely describe the environment of a vehicle subjected to exhaust plume impingement, and the effects of this environment on the vehicle are:

- Description of the thermochemical behavior of the propellant system,
- Prediction of the undisturbed exhaust plume properties via gas dynamic and thermochemical methods,
- Prediction of the body local flow properties for a surface immersed in an exhaust plume,and
- Prediction of heating rates to the surface.

The thermochemical properties are calculated using the NASA-Lewis TRAN72 computer program (Ref. 2-1). The free, undisturbed plume local flow properties are computed using the method of characteristics (Ref. 2-2), with the computations accomplished using the Lockheed-Huntsville Variable O/F Method-of-Characteristics (VOFMOC) computer program (Ref. 2-3). The body local flow properties for a surface immersed in a free plume are calculated using the PLIMP program. Finally the heating rates to the surface are calculated using the surface geometry and impingement properties.

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2.1 THERMOCHEMICAL ANALYSIS

The thermochemical properties of the propellant system being analyzed are computed assuming the combustion products to be chemically frozen or in local chemical equilibrium. The thermochemical properties are obtained by computing the flowfield expansion properties from specified engine combustion chamber conditions using the NASA-Lewis TRAN72 computer program. The TRAN72 program has been modified to generate a binary tape containing all the pertinent parameters needed for flowfield and heat transfer analysis including the mole fractions of each constituent. This method reduces much of the tedious and time-consuming effort normally necessary to prepare input for a single flowfield calculation.

2.2 UNDISTURBED FLOWFIELD ANALYSIS

The VOFMOC program is used to calculate the exhaust plume flow fields. Utilizing the VOFMOC program, the exhaust plume for any arbitrary nozzle and exhaust gas (ideal or reacting) system with gradients in the oxidizer/fuel ratio can be calculated. At finite back pressures the program normally generates the plume flow field using the flow regime continuum methods. As the back pressure is lowered, the outer regions of the plume become progressively more rarefield until eventually the flow becomes free molecular. The VOFMOC program permits transfer of the calculations from a continuum analysis to a free molecular analysis across a "freeze" front in the flow field.

2.3 BODY LOCAL FLOW PROPERTIES ANALYSIS

The PLIMP program is used to calculate body local flow properties for a surface immersed in a free plume. The PLIMP program locates each impinged body and the particular motor with respect to a reference coordinate system. The impinged bodies are then further divided into small elemental areas over which uniform approach flow is assumed. The geometrical subshapes which the computer program can analyze are shown on the following page.

- Conic shapes, i.e., cones, cone frustums, cylinders
- Flat plates, i.e., rectangular, circular, triangular
- Airfoil shapes, and
- Arbitrary axisymmetric bodies whose surface curvature can be described by a polynomial curve fit (maximum of fourth degree).

The local flow conditions on a given elemental area are obtained by locating the centroid of each elemental area in the free plume from a search of the plume flowfield tape. The local impingement angles are obtained using the local flow velocity vector and the elemental area unit-normal vector. Shock calculations are then performed to obtain the local flow properties. An iterative solution is used to perform the equilibrium shock calculations for a real or ideal gas. The real and ideal gas calculations are similar, the difference being that the ideal gas solution converges on the first iteration. A more detailed description of the equilibrium shock calculations is given in Ref. 2.4. Forces and moments are then obtained by numerically integrating the local impact pressure distribution over the body.

Pressures can be calculated in continuum flow using Newtonian theory; modified Newtonian theory; or oblique shock theory; in free molecular flow using kinetic theory; and in transitional flow using empirical relationships.

Convective heating rates in the continuum or transitional flow regimes are calculated (outflow correction theory is not applied). Heating values for continuum-laminar, continuum-turbulent and transitional flow are calculated The method used for the calculation of heat transfer to a body in the transitional regions of the plume has been previously presented in Ref. 2.4. Basically, it amounts to a continuum solution correction to account for the effects of a slip velocity and temperature jump at the surface. Free molecular heating rates are calculated at each elemental area from an energy balance-type equation.

2-3

2.4 STREAMLINE DIVERGENCE ANALYSIS PROGRAMS

The computer programs and calculational techniques described previously have been combined in an automated fashion to fully describe the vehicle environment and the effects of this environment on the vehicle. A schematic flow chart of the calculational process and the flow of information is given in Fig. 2-1. The arrangement in Fig. 2-1 allows the greatest flexibility with a minimum of effort and prevents redundant calculations. For instance, if the heating effects are desired for a previously developed plume, but a new body, then only the PLIMP and Streamline Divergence programs need be rerun.
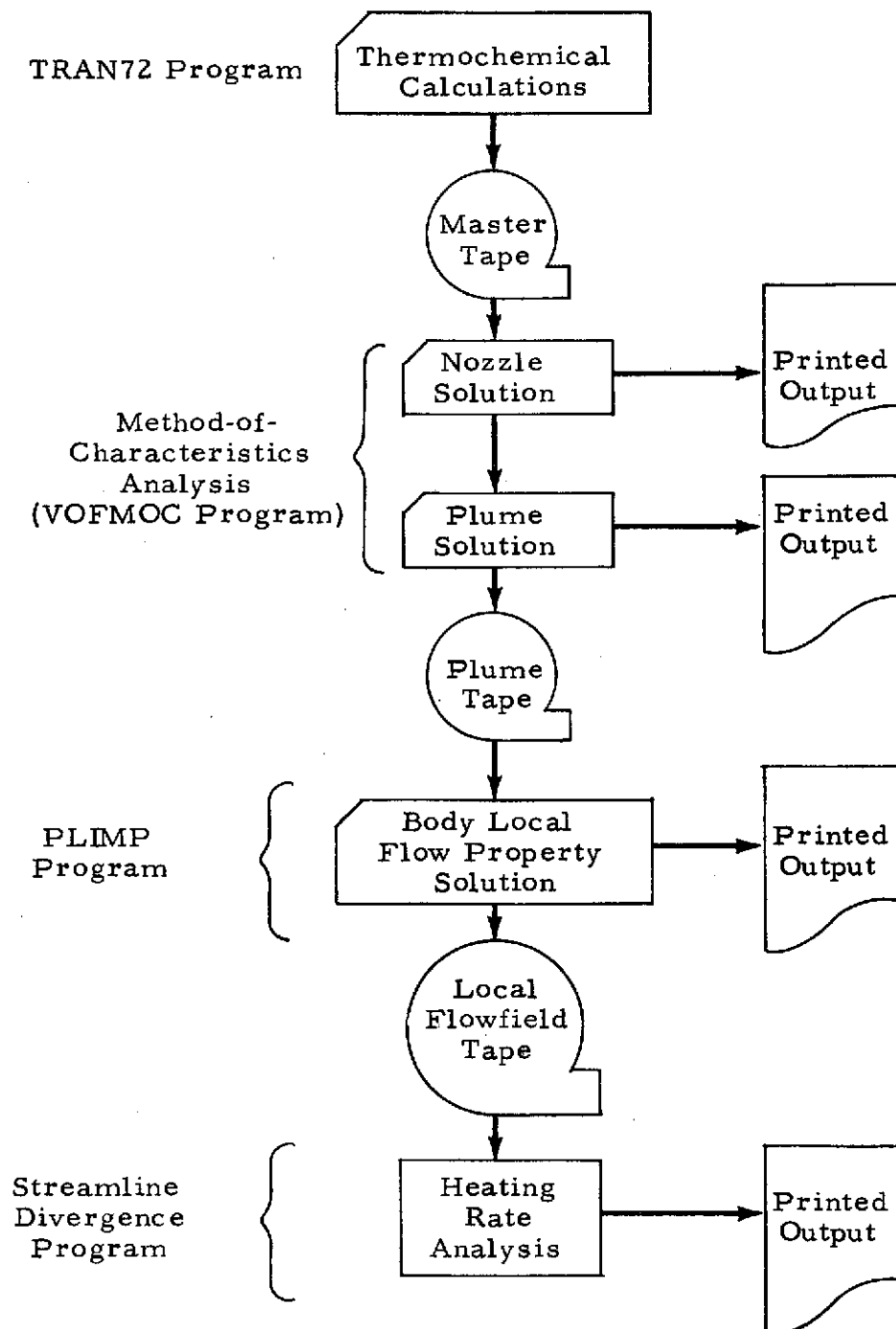
Fig. 2-1 – Calculational Process

Section 3

CALCULATION OF CONVECTIVE HEATING RATES USING A STREAMLINE DIVERGENCE METHOD

This section describes the analytical procedures used by the Stream-line Divergence program. The analysis permits the tracing of inviscid surface streamlines, and the calculation of "outflow corrected" convective heating rates on surfaces subjected to exhaust plume impingement. Basically, the procedure is the following: the Streamline Divergence program applies the Method of the Bicubic Piecewise Polynomial Functions to the impingement pressure distributions calculated by the PLIMP program so that the pressure distribution over the entire body may be described by functional means. The same calculations are applied to the local velocity, local entropy, freestream velocity and freestream entropy distributions determined by the PLIMP program. Once these calculations are performed, the body local flow properties are defined over the entire body in functional terms. The Streamline Divergence program next applies DeJarnette's (Ref. 3-1) streamline divergence methods to determine the streamline patterns and corresponding "equivalent radius" or metric coefficient for the particular body surface subjected to exhaust plume impingement. Convective heating rates for a laminar and turbulent boundary layer are calculated using the integral form of the energy equation. The effects of variable freestream velocity, density and pressure are accounted for through the use of the appropriate transforms of the flat plate solution. Reynolds number is output in order for the user to choose the most applicable of the two convective heating rates that are provided.

3.1 METHOD OF THE BICUBIC PIECEWISE POLYNOMIAL FUNCTIONS

The tracing of inviscid surface streamlines and calculation of convective heating rates are dependent on the surface pressure distribution. Before inviscid surface streamlines and heating rates can be calculated, a description

3-1

of the pressure distribution on the body surface is needed. This pressure distribution must yield continuous first and second derivatives in both the axial and circumferential directions (assuming that a cylindrical coordinate system is used). If the pressure distribution on the body surface could be described by an analytical equation this requirement could easily be met. Unfortunately, this is not the case; most if not all pressure distributions cannot be described by an analytical expression. Often, only the pressure at several points along the circumference of the body surface at several axial stations is known, and the pressure distribution must be generated from these data. Therefore, a numerical technique is required that will generate the pressure distribution from these data, and calculate the required derivatives at "any" point on the body. This technique must also be capable of performing numerical interpolation and differentiation with a reasonable degree of accuracy.

Experience has shown that numerical differentiation methods are generally inaccurate. The method of splines (Ref. 3-2) however, has proven to be an effective method for numerical interpolation, differentiation and integration. The two-dimensional or bicubic spline (which is needed for a three-dimensional axisymmetric body) is able to perform these operations accurately due to the strong convergence properties it possesses.

The Streamline Divergence program applies the Method of the Bicubic Piecewise Polynomial Functions to the pressure distribution determined by the PLIMP program so that the pressure distribution may be described in a functional means. The governing equations, as derived in Ref. 3-2 may be written as follows:

$$P(x, \phi) = C_{00}\left(\frac{1}{\Delta x}\right)\left(\frac{1}{\Delta \phi}\right) + C_{10}\left(\frac{x}{\Delta x}\right)\left(\frac{1}{\Delta \phi}\right) + C_{01}\left(\frac{1}{\Delta x}\right)\left(\frac{\phi}{\Delta \phi}\right)$$

$$+ \dots + C_{33}\left(\frac{x}{\Delta x}\right)^3\left(\frac{\phi}{\Delta \phi}\right)^3 = \sum_{i=0}^{3}\sum_{j=0}^{3} C_{ij}\left(\frac{x^i}{\Delta x}\right)\left(\frac{\phi^j}{\Delta \phi}\right) \tag{3.1}$$

3-2

$$\frac{\partial P}{\partial \phi} (x, \phi) = \sum_{i=0}^{3} \sum_{j=1}^{3} j \, C_{ij} \left(\frac{x}{\Delta x}\right)^{i} \left(\frac{\phi}{\Delta \phi}\right)^{j-1} \tag{3.2}$$

$$\frac{\partial P}{\partial x} (x, \phi) = \sum_{i=1}^{3} \sum_{j=0}^{3} i \, C_{ij} \left(\frac{x}{\Delta x}\right)^{i-1} \left(\frac{\phi}{\Delta \phi}\right)^{j} \tag{3.3}$$

$$\frac{\partial^2 P}{\partial x \, \partial \phi} (x, \phi) = \sum_{i=1}^{3} \sum_{j=1}^{3} i j \, C_{ij} \left(\frac{x}{\Delta x}\right)^{i-1} \left(\frac{\phi}{\Delta \phi}\right)^{j-1} \tag{3.4}$$

$$\frac{\partial^2 P}{\partial \phi^2} (x, \phi) = \sum_{i=0}^{3} \sum_{j=2}^{3} j \, (j-1) \, C_{ij} \left(\frac{x}{\Delta x}\right)^{i} \left(\frac{\phi}{\Delta \phi}\right)^{j-2} \tag{3.5}$$

$$\frac{\partial^2 P}{\partial x^2} (x, \phi) = \sum_{i=2}^{3} \sum_{j=0}^{3} i \, (i-1) \, C_{ij} \left(\frac{x}{\Delta x}\right)^{i-2} \left(\frac{\phi}{\Delta \phi}\right)^{j} \tag{3.6}$$

where:

P   = local flowfield property of interest

C_{ij} = coefficients for the polynonial P (x, ϕ)

and the corresponding coordinate system used is presented in Fig. 3-1. The same equations are applied to the local velocity, local entropy, freestream velocity and freestream entropy distributions determined by the PLIMP program. Once these calculations are performed, the body local flow properties are defined over the entire body in functional terms. The end result obtained from applying the Method of the Bicubic Piecewise Polynomial Functions to
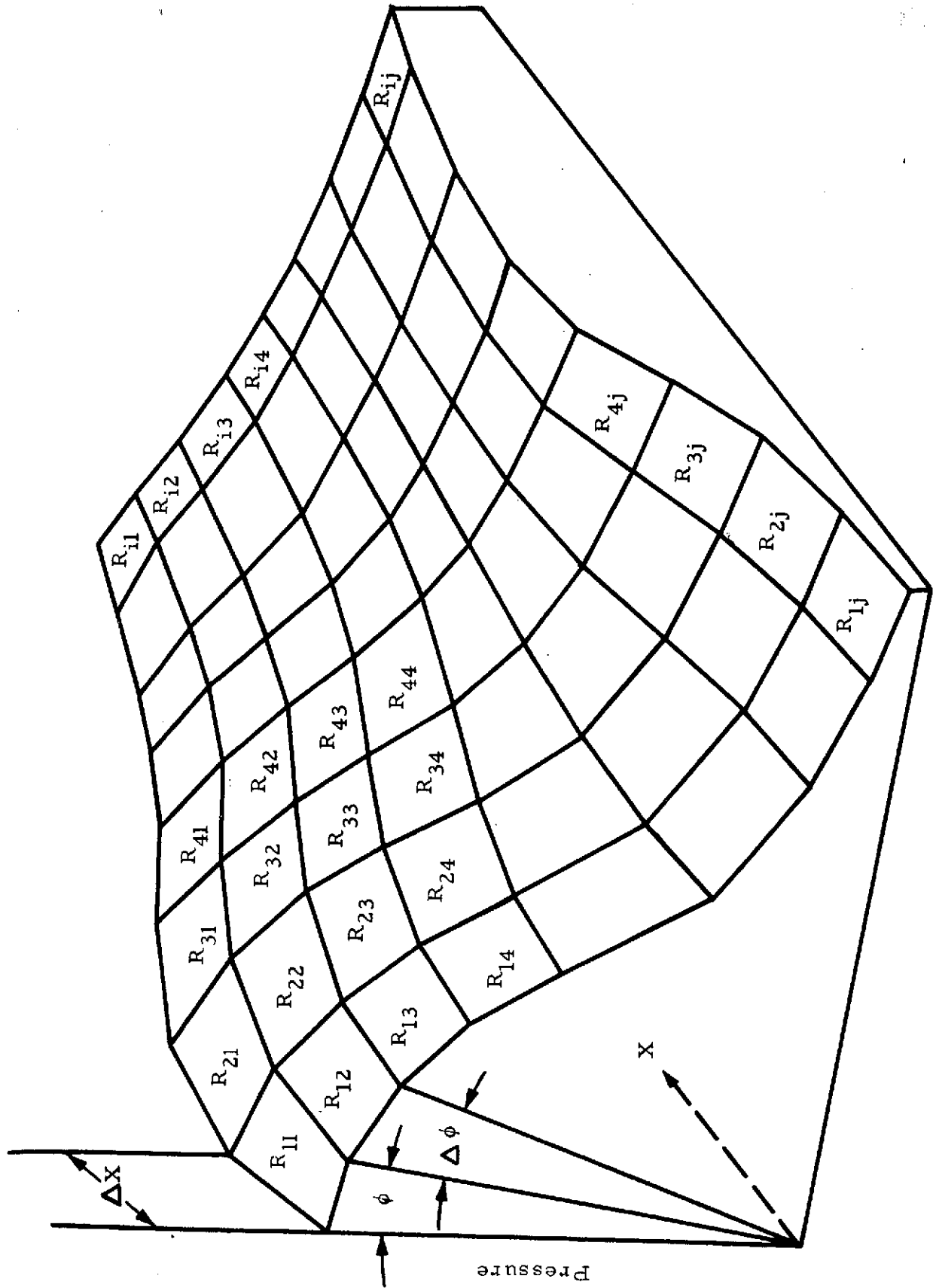
Fig. 3-1 – Coordinate System for Pressure vs X and ϕ

the data presented in Fig. 3-1 is presented in Fig. 3-2. The pressure surface is completely specified by individual sets of coefficients which uniquely describe a bicubic polynomial function in each region over the entire pressure surface. At any point on the surface, the pressure and pressure derivatives can be determined by using Eqs. (3.1) through (3.6) and the corresponding set of coefficients.

Briefly, to determine the value of a particular flowfield property at a point defined by X and ϕ on the body surface, the region R_{ij} which contains x and ϕ is first specified. Equations (3.1) through (3.4) are then evaluated at each of the four corners of the region R_{ij}. These equations make up a set of sixteen simultaneous equations to be solved for the sixteen coefficients describing the particular flowfield property of interest in region R_{ij}. The 16 coefficients are solved for, and then with the value of X and ϕ substituted into Eq. (3.1) to yield the particular flowfield property value.

3.2 THE STREAMLINE DIVERGENCE METHOD

At this point, the body local flowfield properties are defined over the entire body in functional terms. The Streamline Divergence program next applies the methods of DeJarnette (Ref. 3-1) to determine the streamline patterns and calculate the corresponding "equivalent radii" or metric coefficients along each streamline for the particular geometry subjected to exhaust plume impingement. The general form of the equations, as derived in Ref. 3-1 may be written as follows:

$$
\begin{aligned}
\frac{D\theta}{DS} = -\left(\frac{P_s}{\rho V^2}\right) &\left[-\sin\theta\cos\Gamma \frac{\partial}{\partial x}\left(\frac{P}{P_s}\right) \right. \\
&\left. + \frac{(\cos\theta\cos\delta_\phi + \sin\theta\sin\delta_\phi \sin\Gamma)}{f} \frac{\partial}{\partial\phi}\left(\frac{P}{P_s}\right) \right] \\
&- \sin\Gamma \left[\cos\theta\cos\Gamma \frac{\partial\sigma}{\partial x} + \frac{(\sin\theta\cos\delta_\phi - \cos\theta\sin\delta_\phi \sin\Gamma)}{f} \frac{\partial\sigma}{\partial\phi} \right] \quad (3.7)
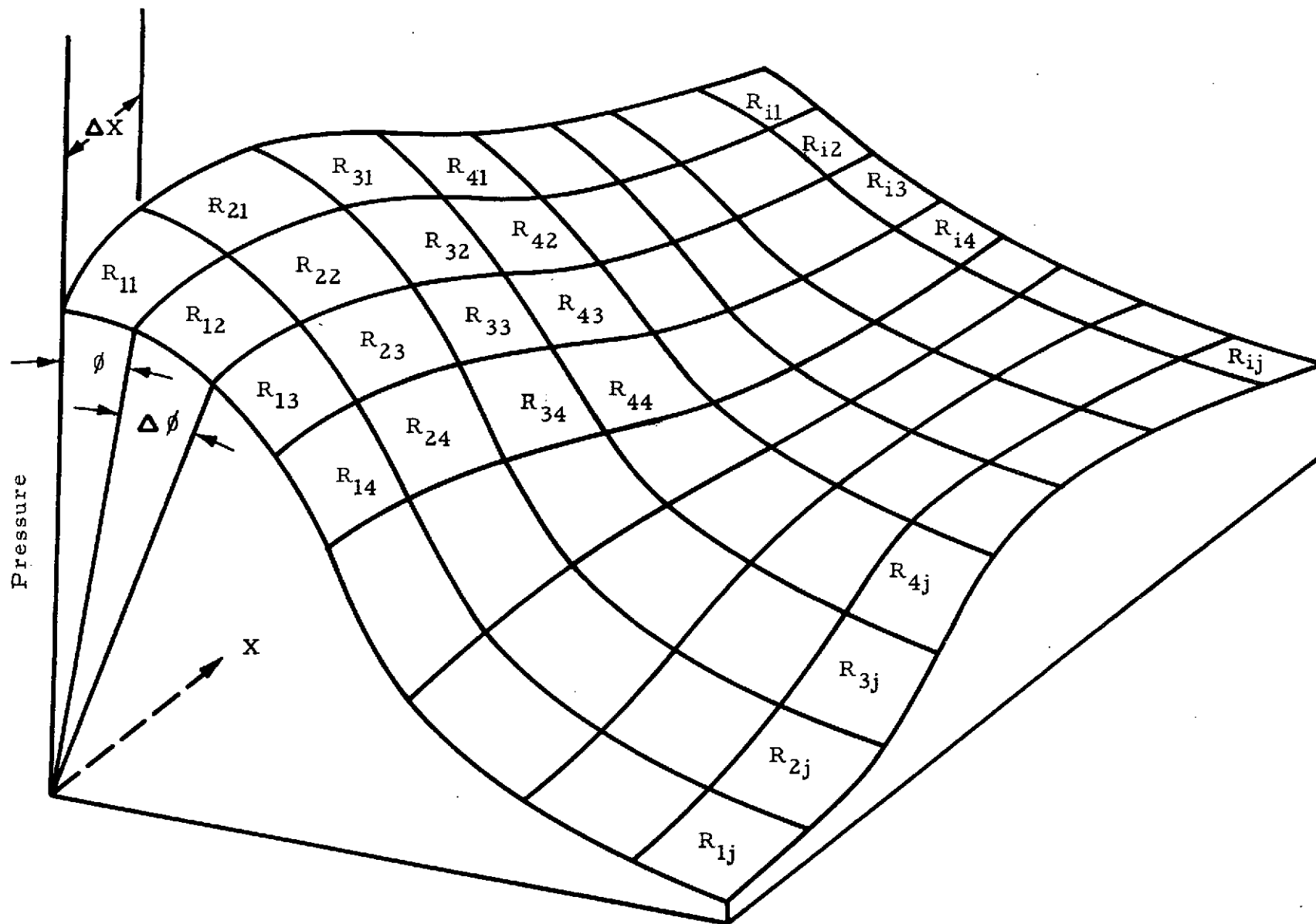\end{aligned}
$$

Fig. 3-2 - Smooth Pressure Surface Fit Generated by the Method of the Bicubic Piecewise Polynomial Functions

$$\frac{Dx}{DS} = \cos\theta \, \cos\Gamma \tag{3.8}$$

$$\frac{D\phi}{DS} = \frac{\sin\theta \, \cos\delta_\phi - \cos\theta \, \sin\delta_\phi \, \sin\Gamma}{f} \tag{3.9}$$

$$\frac{1}{h} \frac{D^2 h}{DS^2} = -\left[\frac{P_s}{\rho V^2} \frac{1}{h} \frac{\partial}{\partial\beta} (P/P_s)\right]^2 (3 - M^2)$$

$$+ \frac{P_s}{\rho V^2} \frac{1}{h} \frac{\partial}{\partial\beta} \left[\frac{1}{h} \frac{\partial}{\partial\beta} (P/P_s)\right]$$

$$+ \frac{\cos^2\Gamma \, \cos\delta_\phi}{f} \left[\frac{\partial\Gamma}{\partial x} \frac{\partial\sigma}{\partial\phi} - \frac{\partial\sigma}{\partial x} \frac{\partial\Gamma}{\partial\phi}\right] \tag{3.10}$$

where

DS = differential arc length along a streamline

P_s = stagnation pressure

ρ = mass density

V = inviscid speed on surface

θ = inclination angle of the inviscid surface streamline

P = static pressure

f = body radius, measured from the longitudinal axis

x = longitudinal coordinate

ϕ = circumferential angle

$$g \equiv \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{1}{f} \frac{\partial f}{\partial\phi}\right)^2 + 1$$

Γ = body angle defined by $\sin\Gamma = \dfrac{\dfrac{\partial f}{\partial x}}{g^{1/2}}$

δ_ϕ = body angle defined by $\sin\delta_\phi = \dfrac{\dfrac{1}{f}\dfrac{\partial f}{\partial \phi}}{\left[1 + \left(\dfrac{1}{f}\dfrac{\partial f}{\partial \phi}\right)^2\right]^{1/2}}$

$\sigma \equiv \phi - \delta_\phi$

β = coordinate normal to the streamline on the body surface

h = "equivalent radius" or metric coefficient in the β direction

M = Mach number

For a cylinder, which is the only geometry currently considered by the Stream-line Divergence program; the angles Γ and δ_ϕ are identically equal to zero, reducing Eqs. (3.7) through (3.10) to:

$$\frac{D\theta}{DS} = \left(\frac{P_s}{V^2}\right)\left[-\sin\theta\,\frac{\partial}{\partial x}(P/P_s) + \frac{\cos\theta}{f}\,\frac{\partial}{\partial\phi}(P/P_s)\right] \qquad (3.11)$$

$$\frac{Dx}{DS} = \cos\theta \qquad (3.12)$$

$$\frac{D\phi}{DS} = \frac{\sin\theta}{f} \qquad (3.13)$$

$$\frac{1}{h}\frac{D^2 h}{DS^2} = -\left[\frac{P_s}{\rho V^2}\left(-\sin\theta\,\frac{\partial(P/P_s)}{\partial x} + \frac{\cos\theta}{f}\,\frac{\partial(P/P_s)}{\partial\phi}\right)\right](3-M^2)$$

$$- \left(\frac{1}{h}\frac{Dh}{Ds}\right)\left(\frac{P_s}{\rho V^2}\right)\left(\cos\theta\,\frac{\partial(P/P_s)}{\partial x} - \frac{\sin\theta}{f}\,\frac{\partial(P/P_s)}{\partial\phi}\right)$$

$$+ \frac{P_s}{\rho V^2}\left(\frac{1}{f}\sin\theta\cos\theta\,\frac{\partial(P/P_s)}{\partial\phi} + \sin^2\theta\,\frac{\partial^2(P/P_s)}{\partial x^2}\right.$$

$$\left. - \frac{2}{f}\sin\theta\cos\theta\,\frac{\partial^2 \rho}{\partial x\partial\phi} + \frac{\cos^2\theta}{f^2}\,\frac{\partial^2(P/P_s)}{\partial\phi}\right) \qquad (3.14)$$

respectively. It should be noted at this point that other geometries can easily be incorporated into the Streamline Divergence program simply by making the proper substitutions into Eqs. (3.7) through (3.10) to yield new applicable Eqs. (3.11) through (3.14).

The values of local static pressure, velocity and the derivatives

$$\frac{\partial P}{\partial x} \ , \ \frac{\partial P}{\partial \phi} \ , \ \frac{\partial^2 P}{\partial x^2} \ , \ \frac{\partial^2 P}{\partial \phi^2} \ \text{and} \ \frac{\partial^2 P}{\partial x \partial \phi}$$

are determined by using the bicubic piecewise polynomial functions at the correct geometric location on the body surface. The inviscid surface stream-lines and their corresponding metric coefficient, h, are calculated by numer-ically integrating Eqs. (3.11) through (3.14) simultaneously. The numerical integration scheme used is the fourth order variable step size Runge-Kutta method. The initial conditions required to start the integration of each streamline are determined external to the program and input by means of punched data cards.

3.3 RELATIONSHIP OF THE METRIC COEFFICIENTS TO CONVECTIVE HEAT TRANSFER THEORY

The convective heating rates for a laminar and turbulent boundary layer are calculated using the integral form of the energy equation. The effects of variable freestream velocity, density and pressure are accounted for through the use of the appropriate transforms of the flat plate solution. Non-constant properties through the boundary layer are also accounted for.

In Ref. 3-3 (among others) it is shown that the energy equation can be reduced to an equation with enthalpy as the dependent variable under a number of different conditions including some cases where there is dissociation, chemical reaction and mass diffusion within the boundary layer. The com-plete development of the enthalpy form of the equation and the evaluation of the surface heat rate are presented in Ref. 3-3.

Based on examination of several "exact" laminar boundary layer solutions, Eckert (Ref. 3-4) recommends that the effects of variable gas properties through the boundary layer can be accounted for by simply evaluating the properties at a "reference enthalpy" and using these values in the constant property solutions as obtained by Blasius. Based on this method, the convective heating rate to the wall is evaluated using:

Laminar Boundary Layer

$$\dot{q} = \frac{0.332}{Pr^{*\,2/3}} \; \frac{(\rho^* \mu^* V_e)^{0.5}}{X_L^{.5}} \; (H_r - h_w) \tag{3.15}$$

where: X_L = laminar characteristic running length. X_L is obtained for variable property flow over an arbitrary shape, by numerically integrating the following equation along a flowfield streamline, (Ref. 3-3)

$$X_L = \frac{1}{\rho^* \mu^* V_e h^2} \int_o^S \rho^* \mu^* V_e h^2 \, dS$$

H_r = adiabatic wall enthalpy

$$H_r = h_\infty + \frac{V_\infty^2}{2} - \frac{V_e^2}{2} (1 - r)$$

r = recovery factor

$$r = \sqrt{P_r^*}$$

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Turbulent Boundary Layer

$$\dot{q} = \frac{0.0292}{\left[1 + .212 \left(Re_T\right)^{-.1} (P_r^* - 1)\right]} (\rho^* V_e)^{0.8} \left(\frac{\mu^*}{X_T}\right)^{.2} (H_r - h_w) \qquad (3.16)$$

where: X_T = turbulent characteristic running length

$$X_T = \frac{1}{\rho^* \mu^* V_e h^{1.25}} \int_0^S \rho^* \mu^* V_e h^{1.25} \, dS$$

Re_T = turbulent Reynolds number

$$Re_T = \rho^* V_e X_T / \mu^*$$

$r \quad = P_r^{* \ 1/3}$

$P_r^* \quad$ = Prandtl number

$\mu^* \quad$ = absolute viscosity

$V_e \quad$ = velocity at the edge of the boundary layer

$V_\infty \quad$ = freestream velocity

$h_w \quad$ = wall enthalpy

$h_\infty \quad$ = freestream enthalpy

$S \quad$ = wetted length along the streamline

$h \quad$ = metric coefficient

The starred (*) properties refer to properties evaluated at a temperature corresponding to a "reference enthalypy," H^*, where

3-11

$$H^* = h_e + 0.5(H_o - h_e) + 0.22(H_r - h_e)$$

h_e = static enthalpy at the boundary layer edge

H_o = stagnation enthalpy

The wall enthalpy (h_w) in Eqs. (3.15) and (3.16) may be calculated assuming either an infinitely fast recombination rate in the boundary layer (chemical equilibrium) or an infinitely slow recombination rate (chemically frozen). The upper extreme to the convective heating rate is determined by assuming equilibrium chemistry throughout the boundary layer and a fully catalytic wall. The lower extreme to the convective heating rate is determined by assuming a frozen thermal boundary layer and a non-catalytic wall.

Eckert's reference enthalpy method has been chosen for use in the above analysis for demonstration purposes. Other methods do exist (though they have not been evaluated under the present study) that may be applied to the problem of exhaust plume impingement and, can be incorporated into the computer code with a minimum of programming difficulty.

Section 4
USER'S INPUT GUIDE FOR THE STREAMLINE
DIVERGENCE COMPUTER PROGRAM

The Streamline Divergence program utilizes three forms of input: (1) data statements within the program; (2) magnetic tapes; and (3) punched data cards. Data statements are used to input thermodynamic data (for reaction products in functional form) that seldom change. Magnetic tapes and punched data cards are used to input data that frequently vary from one exhaust plume flow field to another or from run to run. The program magnetic tape assignments are given in Table 4-1. Each punched data card and its use is explained in Section 4.1.

4.1 PROGRAM INPUT GUIDE

Card 1: Program Control Card

Format	Column	Item
16I5	5	NFIT, number of flowfield properties to be curve fit using the Method of Bicubic Piecewise Polynomial Functions (upper limit of 5)
	10	IUNIT, tape unit number on which impingement data are written
	15	NOXCUT, number of axial printout intervals
	20	NOYCUT, number of angular printout intervals
	25	JMAX, number of points on each streamline where heating rates are to be calculated (upper limit of 150)
	30	ICARD, number of species data input on punched data cards
	35	NPRINT, control option that indicates to print (=1) or not to print (=0) intermediate data (used during program checkout procedures)

Table 4-1

MAGNETIC TAPE ASSIGNMENTS FOR THE STREAMLINE
DIVERGENCE PROGRAM

Where Required	Tape Units U-1108	Tape Unit Function
<u>Section 1</u> Subroutine PLIRED Subroutine PLIRED is the controlling routine which arranges the PLIMP output in the form used by the data acquisition routines. This subroutine is called once at the beginning of each run.	IUNIT	Impingement data generated by the PLIMP program (input data)
<u>Section 2</u> Convective Heating Rate Analysis This section performs the convective heating rate analysis along each inviscid surface streamline being traced.	3	Thermochemical properties data generated by the TRAN72 program (input data)

Card 2:

Format	Column	Item
16I5	5	NOSL, number of individual streamlines to be traced (upper limit of 10)

Card 3: This card contains the initial conditions required to start the tracing of each streamline utilizing the fourth order variable step-size Runge-Kutta method. There is one card for each streamline to be traced.

6F10.4	1-10	XSTART, initial axial location of streamline (in.)
	11-20	YSTART, initial angular location of streamline (rad)
	21-30	TSTART, initial streamline direction (rad)
	31-40	HSTART, initial metric coefficient (in.)
	41-50	ASTART, initial derivative of metric coefficient with respect to streamline coordinate (in./in.)
	51-60	DELS, integration step size along streamline (in.)

Card 4:

10E11.4	1-11	TWALL, body surface wall temperature. Assumed to be constant over the entire body (^{O}R)

Card 5:

16I5	5	NENTLP, control option that indicates to calculate wall enthalpy assuming frozen (=1) or equilibrium (=2) chemistry throughout the boundary layer.

Card(s) 6: Species concentration cards (used when NENTLP = 2, and ICARD > 0). This card(s) specifies the mole fractions of the flowfield gas composition.

4(A6, 4X, E10.6)	1-6 21-26 41-46 61-66	Chemical species name, i.e., H_2, H_2O
	11-20 31-40 51-60 71-80	Mole fraction

The species names are left-adjusted.

Cards 7, 8 and 9:   Wall enthalpy array cards (used when NENTLP = 1). These cards are used to define wall enthalpy as a function of impingement pressure and body surface wall temperature.

Card 7:

Format	Column	Item
5I5	5	NPWALL, number of independent pressures in the impingement pressure array
	10	NTWALL, number of independent temperatures in the body surface wall temperature array

Card(s) 8 and 9:

E10.6/
(8E10.6)

Card 8:

1-10		TWALL, temperature in body surface wall temperature array (^{o}R)

Card(s) 9:

1-10		PWALL, pressure in the impingement pressure
21-30		array (psi)
41-50		
61-70		
11-20		HWALL, corresponding dependent wall enthalpy
31-40		(Btu/lbm)
51-60		
71-80		

Cards 8 and 9 are repeated NTWALL times.

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Card 10: This card contains identifying information for the particular problem of interest. This information is printed at the top of each page of the output.

Format	Column	Item
12A6, I5	1-72	Comment card; header information such as problem title may go in these columns.
	73-77	Case number of the particular problem being analyzed.

Section 5

PROGRAM NOMENCLATURE

The Streamline Divergence Program, like all computer programs, contains its own particular system of terms. This section presents a list of all "key" nomenclature used in the program, and may be used as a quick reference when working with the program.

ADSTART	INITIAL FIRST DERIVATIVE OF METRIC COEFFICIENT WITH RESPECT TO DISTANCE ALONG A STREAMLINE (REQUIRED TO START THE TRACING OF EACH STREAMLINE)
DADS	SECOND DERIVATIVE OF METRIC COEFFICIENT WITH RESPECT TO DISTANCE ALONG A STREAMLINE
DELS	INTEGRATION STEP SIZE ALONG A STREAMLINE (INCHES)
DHDS	FIRST DERIVATIVE OF METRIC COEFFICIENT WITH RESPECT TO DISTANCE ALONG A STREAMLINE
DTDS	FIRST DERIVATIVE OF STREAMLINE DIRECTION WITH RESPECT TO DISTANCE ALONG A STREAMLINE
DXDS	FIRST DERIVATIVE OF X WITH RESPECT TO DISTANCE ALONG A STREAMLINE
DYDS	FIRST DERIVATIVE OF PHI WITH RESPECT TO DISTANCE ALONG A STREAMLINE
EAW	LAMINAR ADIABATIC WALL ENTHALPY (BTU/LBM)
EAWT	TURBULENT ADIABATIC WALL ENTHALPY (BTU/LBM)
EW	WALL ENTHALPY (BTU/LBM)
HSTART	INITIAL METRIC COEFFICIENT OF STREAMLINE (REQUIRED TO START THE TRACING OF EACH STREAMLINE) (INCHES)
HE	ENTHALPY AT THE EDGE OF THE BOUNDARY LAYER (BTU/LBM)
HWALL	ENTHALPY IN THE WALL ENTHALPY ARRAY (BTU/LBM)
H2	METRIC COEFFICIENT (INCHES)
ICARD	NUMBER OF SPECIE DATA INPUT ON PUNCHED DATA CARDS
IUNIT	TAPE UNIT NUMBER ON WHICH IMPINGEMENT DATA IS WRITTEN
JMAX	NUMBER OF POINTS ON EACH STREAMLINE WHERE HEATING RATES ARE TO BE CALCULATED
NOLX	COUNTER THAT INDICATES THE NUMBER OF THE INVISCID SURFACE STREAMLINE PRESENTLY BEING TRACED
NENTLP	CONTROL OPTION THAT INDICATES TO CALCULATE WALL ENTHALPY ASSUMING FROZEN (=1) OR EQUILIBRIUM (=2) CHEMISTRY THROUGHOUT THE BOUNDARY LAYER
NFIT	NUMBER OF FLOW FIELD PROPERTIES TO BE CURVE FIT USING THE METHOD OF BICUBIC PIECEWISE POLYNOMIAL FUNCTIONS
NOSL	NUMBER OF INDIVIDUAL STREAMLINES TO BE TRACED
NOXCUT	NUMBER OF AXIAL PRINT OUT INTERVALS
NOYCUT	NUMBER OF ANGULAR PRINT OUT INTERVALS

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NPRINT	CONTROL OPTION THAT INDICATES TO PRINT (=1) OR NOT TO PRINT (=0) INTERMEDIATE DATA
NPWALL	NUMBER OF INDEPENDENT PRESSURES IN THE IMPINGEMENT PRESSURE ARRAY
NTWALL	NUMBER OF INDEPENDENT TEMPERATURES IN THE BODY SURFACE WALL TEMPERATURE ARRAY
PHI	ANGULAR LOCATION ON THE BODY SURFACE (RADIANS)
PR	LOCAL PRANDTL NUMBER
PRS	REFERENCE LAMINAR LOCAL PRANDTL NUMBER
PRST	REFERENCE TURBULENT LOCAL PRANDTL NUMBER
PW	LOCAL IMPINGEMENT PRESSURE (PSF)
PWALL	PRESSURE IN THE IMPINGEMENT PRESSURE ARRAY (PSI)
QL	LAMINAR CONVECTIVE HEATING RATE (BTU/FT**2 SEC)
QT	TURBULENT CONVECTIVE HEATING RATE (BTU/FT**2 SEC)
RELAM	LAMINAR REYNOLDS NUMBER
RETURB	TURBULENT REYNOLDS NUMBER
RHO	LOCAL DENSITY (LBM/FT**3)
RHOS	REFERENCE LAMINAR LOCAL DENSITY (LBM/FT**3)
RHOST	REFERENCE TURBULENT LOCAL DENSITY (LBM/FT**3)
S	LOCAL ENTROPY (BTU/LBM °R)
SINF	FREE STREAM ENTROPY (BTU/LBM °R)
TE	LOCAL FLOW FIELD TEMPERATURE AT THE EDGE OF THE BOUNDARY LAYER (°R)
TK	BODY SURFACE WALL TEMPERATURE (°K)
TSTART	INITIAL STREAMLINE DIRECTION (REQUIRED TO START THE TRACING OF EACH STREAMLINE) (RADIANS)
TW	BODY SURFACE WALL TEMPERATURE (°R)
TWALL	BODY SURFACE WALL TEMPERATURE. ASSUMED TO BE CONSTANT OVER THE ENTIRE BODY (°R)
U	LOCAL VELOCITY (FT/SEC)
V(A,B,C,D)	ARRAY USED TO STORE LOCAL FLOW PROPERTIES AND THEIR DERIVATIVES AS A FUNCTION OF GEOMETRIC LOCATION ON THE BODY SURFACE
V(-,-,C,-)	DEFINES THE AXIAL GEOMETRIC LOCATION ON THE BODY SURFACE (INCHES)
V(-,-,-,D)	DEFINES THE ANGULAR GEOMETRIC LOCATION ON THE BODY SURFACE (RADIANS)

V(-,B,-,-)   DEFINES THE DERIVATIVE OF V WITH RESPECT TO X (B=2), WITH RESPECT
              TO PHI (B=3), OR WITH RESPECT TO X AND PHI (B=4). IF B =1, NO
              DERIVATIVE HAS BEEN TAKEN

V(A,-,-,-)   DEFINES THE LOCAL FLOW PROPERTY. LOCAL VELOCITY (A=1),LOCAL ENTROPY
              (A=2), FREE STREAM VELOCITY (A=3), FREE STREAM ENTROPY (A=4), AND
              LOCAL IMPINGEMENT PRESSURE (A=5)

VIS   ABSOLUTE VISCOSITY (LBM/FT SEC)

VINF   FREE STREAM VELOCITY (FT/SEC)

VISS   REFERENCE LAMINAR ABSOLUTE VISCOSITY (LBM/FT SEC)

VISST   REFERENCE TURBULENT ABSOLUTE VISCOSITY (LBM/FT SEC)

VMAX(1)   MAXIMUM LOCAL VELOCITY ON THE BODY SURFACE (FT/SEC)

VMAX(2)   MAXIMUM LOCAL ENTROPY ON THE BODY SURFACE (BTU/LBM °R)

VMAX(3)   MAXIMUM FREE STREAM VELOCITY (FT/SEC)

VMAX(4)   MAXIMUM FREE STREAM ENTROPY (BTU/LBM °R)

VMAX(5)   MAXIMUM LOCAL IMPINGEMENT PRESSURE ON THE BODY SURFACE (PSF)

X   AXIAL LOCATION ON THE BODY SURFACE (INCHES)

XLLAM   LAMINAR CHARACTERISTIC RUNNING LENGTH (INCHES)

XLTURB   TURBULENT CHARACTERISTIC RUNNING LENGTH (INCHES)

XM   LOCAL MACH NUMBER

XSTART   INITIAL AXIAL LOCATION OF STREAMLINE (REQUIRED TO START THE TRACING
              OF EACH STREAMLINE) (INCHES)

XX1   AXIAL LOCATION ON THE BODY SURFACE (INCHES)

YDOT(1)   THE STORED VALUE OF DXDS

YDOT(2)   THE STORED VALUE OF DYDS

YDOT(3)   THE STORED VALUE OF DTDS

YDOT(4)   THE STORED VALUE OF DADS

YDOT(5)   THE STORED VALUE OF DHDS

YSTART   INITIAL ANGULAR LOCATION OF STREAMLINE (REQUIRED TO START THE
              TRACING OF EACH STREAMLINE) (RADIANS)

YY1   ANGULAR LOCATION ON THE BODY SURFACE (RADIANS)

Section 6

USE OF THE STREAMLINE DIVERGENCE PROGRAM

To familiarize the user with the operation of the program, a sample
case is presented. The user is guided through a step-by-step procedure
in setting up the data, coding the input for the program and interpreting the
output from the program.

6.1 SAMPLE PROBLEM

A typical shuttle configuration is chosen as the example. In this ex-
ample, one is interested in tracing inviscid surface streamlines, and calcu-
lating "outflow corrected" convective heating rates on that portion of the
booster fuselage subjected to exhaust plume impingement. The overall coor-
dinate system and the orientation between the booster and the orbiter engine
is shown in Fig. 6-1. The coordinate system reference point corresponds to
the first point analyzed by the PLIMP program. The upper surface of the
booster fuselage was mathematically modeled as a cylinder (radius of 5.805
in.).

6.2 IDENTIFICATION OF THE INPUT/OUTPUT DATA

The input data for the problem configuration described in Section 6.1
will be illustrated by identifying the computer printout of the input data in
Table 6-1.

The input data for the configuration of Section 6.1 is identified as
follows:

(1)   This is an identification of the problem, and is printed at
the top of each page of data. This information is contained
in card 10 of the input.

Fig. 6-1 - Overall Coordinate System and Orientation Between the Booster and Orbiter Engine
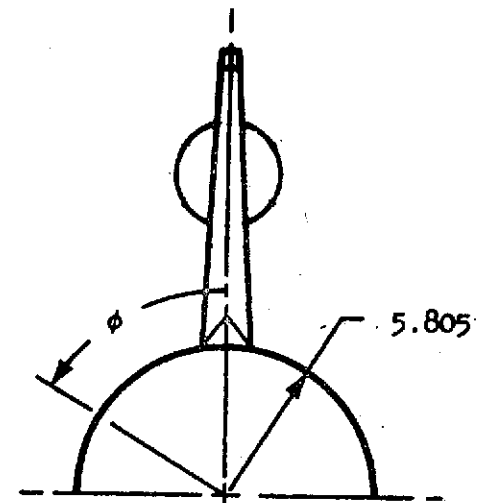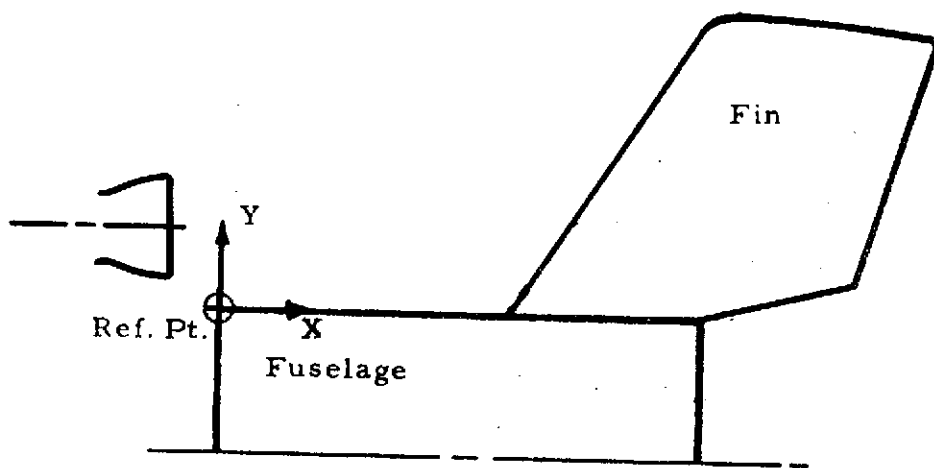
Table 6-1

STREAMLINE DIVERGENCE PROGRAM OUTPUT LISTING

CASE NO.   1
(1)   LOCKHEED/HUNTSVILLE STREAMLINE DIVERGENCE COMPUTER PROGRAM SAMPLE OUTPUT

(2)	NFIT	IUNIT	NOXCUT	NOYCUT	JMAX		
	5	11	20	25	145		

	STREAMLINE	XSTART	YSTART	TSTART	HSTART	ASTART	DELS
(3)	1	3.0600	.0000	.0500	5.8050	.0000	.1000
	2	3.0600	.0000	.1000	5.8050	.0000	.1000
	3	3.0600	.0000	.1500	5.8050	.0000	.1000
	4	3.0600	.0000	.2000	5.8050	.0000	.1000
	5	3.0600	.0000	.2500	5.8050	.0000	.1000

	TWALL	NENTLP	ICARD
(4)	540.0000	2	0

TOTAL FREESTREAM ENTHALPY =   -.8667+00 BTU/LBM

Table 6-1 (Continued)

CASE NO.   1   PAGE   2
LOCKHEED/HUNTSVILLE STREAMLINE DIVERGENCE COMPUTER PROGRAM SAMPLE OUTPUT

STREAMLINE	X (IN)	PHI (RAD)	VELOCITY (FT/SEC)	ENTROPY (FT**2/SEC**2/R)	MACH	RHO (LBM/CF)	HAR ----------(BTU/LBM)----------	HART	HW	TE (DEG R)
	PW (PSF)	VISW (LBM/FT/SEC)	PRW	EQUIV R ----------(IN)----------	XLLAM	XLTURB	QL (BTU/FT**2/SEC)	QT	RELAM ----DIMENSIONLESS----	RETURB

(Data columns labeled with circled numbers 5 through 25)

(5) 1	(6) .3260+01	(7) .1748-02	(8) .1232+05	(9) .2039+05	(10) .2981+01	(11) .1383-04	(12) -.6980+03	(13) -.4806+03	(14) -.5553+04	(15) .4061+04
	(16) .6423+01	(17) .5092-04	(18) .5595+00	(19) .5817+01	(20) .1631+00	(21) .1628+00	(22) .2780+02	(23) .4772+01	(24) .1579+02	(25) .1700+02
1	.3459+01	.3541-02	.1262+05	.1876+05	.3103+01	.1734-04	-.7282+03	-.5097+03	-.5553+04	.3919+04
	.7785+01	.4966-04	.5663+00	.5853+01	.2799+00	.2748+00	.2575+02	.5866+01	.4048+02	.4378+02
1	.3659+01	.5376-02	.1289+05	.1727+05	.3221+01	.2108-04	-.7561+03	-.5291+03	-.5553+04	.3788+04
	.9036+01	.4851-04	.5729+00	.5911+01	.3581+00	.3591+00	.2664+02	.7373+01	.7703+02	.8378+02
1	.3859+01	.7248-02	.1313+05	.1594+05	.3337+01	.2507-04	-.7816+03	-.5467+03	-.5553+04	.3647+04
	.1018+02	.4702-04	.5786+00	.5984+01	.4295+00	.4291+00	.2844+02	.9159+01	.1285+03	.1408+03
1	.4059+01	.9155-02	.1335+05	.1475+05	.3457+01	.2919-04	-.8075+03	-.5626+03	-.5553+04	.3499+04
	.1121+02	.4523-04	.5835+00	.6082+01	.4554+00	.4929+00	.3259+02	.1114+02	.1969+03	.2187+03
1	.4258+01	.1109-01	.1354+05	.1370+05	.3567+01	.3307-04	-.8249+03	-.5766+03	-.5553+04	.3368+04
	.1213+02	.4365-04	.5877+00	.6190+01	.5572+00	.5552+00	.3152+02	.1321+02	.2937+03	.3208+03
1	.4458+01	.1307-01	.1371+05	.1279+05	.3668+01	.3662-04	-.8416+03	-.5888+03	-.5553+04	.3251+04
	.1295+02	.4228-04	.5914+00	.6310+01	.6145+00	.6182+00	.3309+02	.1532+02	.4064+03	.4495+03
1	.4658+01	.1507-01	.1385+05	.1198+05	.3762+01	.3984-04	-.8571+03	-.5997+03	-.5553+04	.3148+04
	.1366+02	.4108-04	.5947+00	.6442+01	.6723+00	.6821+00	.3446+02	.1743+02	.5425+03	.6068+03
1	.4857+01	.1713-01	.1398+05	.1127+05	.3849+01	.4274-04	-.8700+03	-.6086+03	-.5570+04	.3056+04
	.1427+02	.4002-04	.5975+00	.6584+01	.7423+00	.7589+00	.3514+02	.1925+02	.7043+03	.7960+03
1	.5057+01	.1914-01	.1409+05	.1062+05	.3933+01	.4538-04	-.8823+03	-.6173+03	-.5570+04	.2973+04
	.1478+02	.3907-04	.6031+00	.6735+01	.7973+00	.8213+00	.3629+02	.2132+02	.8864+03	.1012+04
1	.5257+01	.2124-01	.1420+05	.1002+05	.4006+01	.4781-04	-.8936+03	-.6251+03	-.5570+04	.2897+04
	.1519+02	.3820-04	.6024+00	.6894+01	.8507+00	.8824+00	.3740+02	.2343+02	.1094+04	.1259+04
1	.5456+01	.2335-01	.1430+05	.9475+04	.4083+01	.5004-04	-.9042+03	-.6324+03	-.5570+04	.2826+04
	.1552+02	.3741-04	.6346+00	.7061+01	.9025+00	.9425+00	.3846+02	.2556+02	.1327+04	.1541+04
1	.5656+01	.2549-01	.1438+05	.8967+04	.4149+01	.5205-04	-.9136+03	-.6391+03	-.5570+04	.2761+04
	.1577+02	.3668-04	.6366+00	.7236+01	.9537+00	.1012+01	.3945+02	.2769+02	.1587+04	.1858+04
1	.5855+01	.2765-01	.1447+05	.8499+04	.4216+01	.5391-04	-.9225+03	-.6453+03	-.5570+04	.2700+04
	.1593+02	.3600-04	.6384+00	.7416+01	.1005+01	.1063+01	.4036+02	.2979+02	.1874+04	.2210+04
1	.6055+01	.2984-01	.1454+05	.8068+04	.4280+01	.5532-04	-.9308+03	-.6511+03	-.5570+04	.2644+04
	.1602+02	.3538-04	.6113+00	.7603+01	.1057+01	.1124+01	.4115+02	.3183+02	.2188+04	.2599+04
1	.6255+01	.3205-01	.1461+05	.7671+04	.4342+01	.5654-04	-.9345+03	-.6564+03	-.5570+04	.2591+04
	.1604+02	.3479-04	.6116+00	.7797+01	.1109+01	.1187+01	.4183+02	.3381+02	.2528+04	.3023+04

② These are the control variables used in the inviscid surface streamline tracing section of the program. These data are contained in card 1 of the input.

③ These are the initial conditions required to start the tracing of each streamline utilizing the fourth order Runge-Kutta method.   These data are contained in cards 2 and 3 of the input.

④ These are the control variables used in the convective heating rate section of the program. These data are contained in cards 1, 4 and 5 of the input.

The output of the program is identified by the symbols ⑤ through ㉕ . These are the body local flow properties, heating rates, and characteristic dimensions calculated along each inviscid surface streamline. The variables are defined as follows:

⑤ Indicates the number of the inviscid surface streamline presently being traced.

⑥ Axial location on the body surface

⑦ Angular location on the body surface

⑧ Local velocity

⑨ Local entropy

⑩ Local Mach number

⑪ Local density

⑫ Laminar adiabatic wall enthalpy

⑬ Turbulent adiabatic wall enthalpy

⑭ Local wall enthalpy

⑮ Local flowfield temperature at the edge of the boundary layer

⑯ Local impingement pressure

⑰ Local viscosity

⑱ Local Prandtl number

6-5

(19)   Local metric coefficient

(20)   Laminar characteristic running length

(21)   Turbulent characteristic running length

(22)   Laminar convective heating rate

(23)   Turbulent convective heating rate

(24)   Laminar Reynolds number

(25)   Turbulent Reynolds number

6.3   SAMPLE RESULTS

Some of the pertinent data contained in Section 6.2 is presented graphically in this section. The impingement pressure distribution and other local flowfield properties used by the program were calculated by the Lockheed-Huntsville PLIMP program. Figure 6-2 is a plot of inviscid surface streamline coordinates (ϕ versus X) for several initial streamline starting angles, θ. Figure 6-3 is a plot of metric coefficient versus distance X for one of these streamlines ($\theta = .05$ radians). In Fig. 6-2, the streamlines diverge at an increasing rate as they proceed axially down the cylinder. The rate of divergence at any point on the cylinder increases with increasing initial starting angle, θ. This divergence, i.e., the spreading of the streamlines, is reflected by the metric coefficient plot in Fig. 6-3. Figure 6-4 is a plot of the laminar impingement heating rate distribution along the streamline with an initial starting angle, $\theta = .05$ radians.

6-6

LMSC-HREC TM D390450
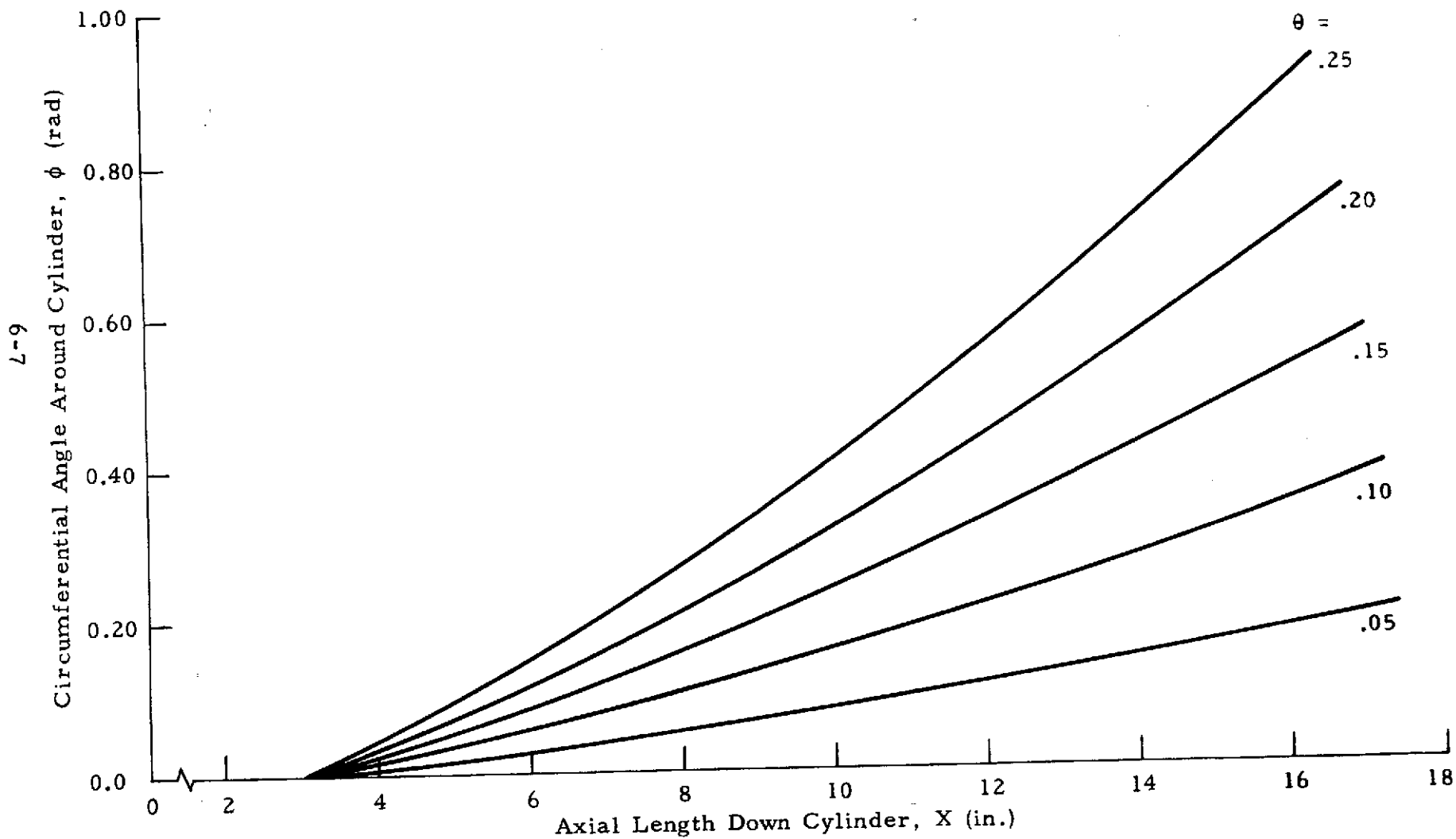
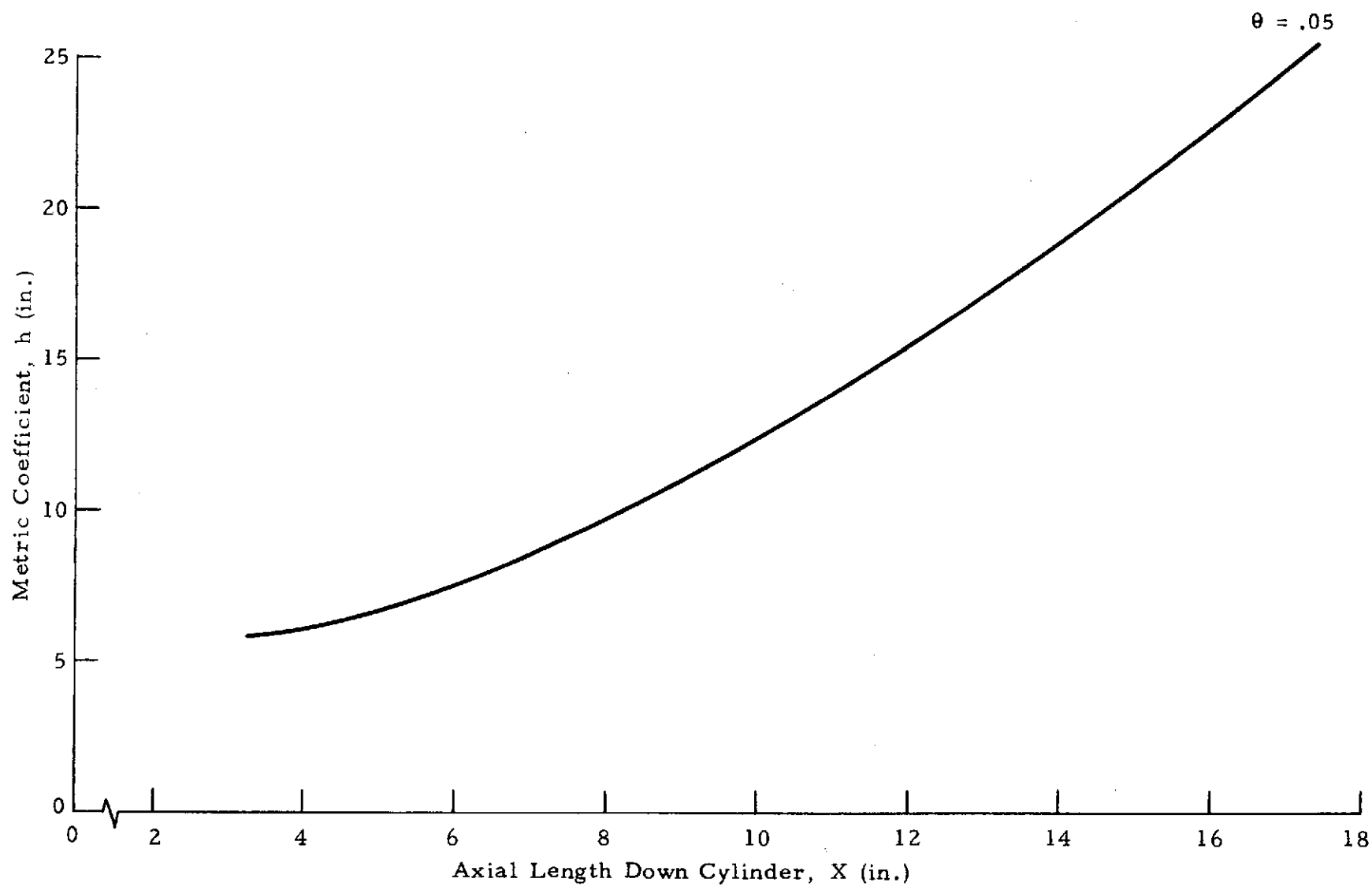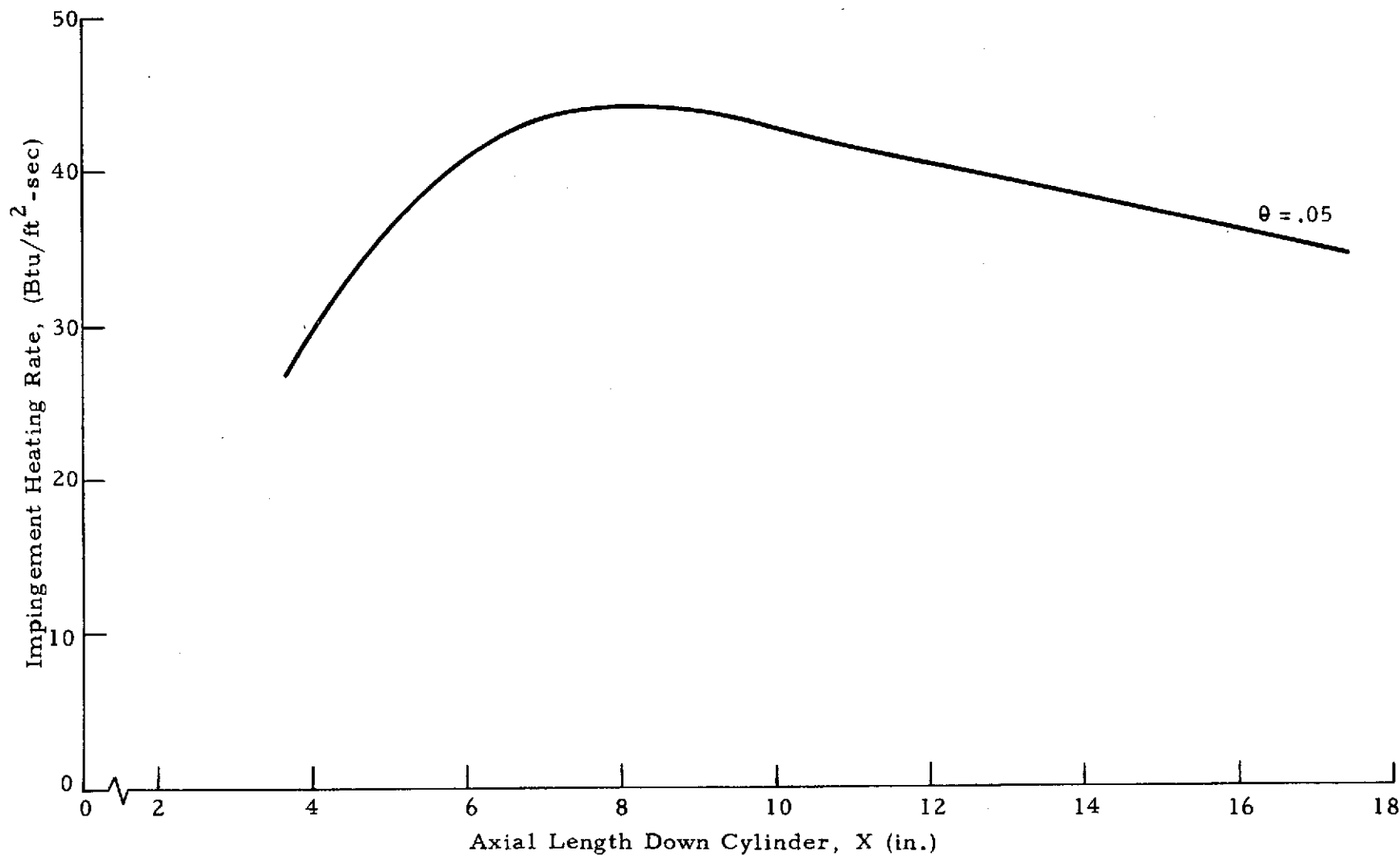Fig. 6-2 - Streamline Coordinates Plot for Various Initial Starting Angles, θ

Fig. 6-3 - Metric Coefficient Variation Along a Streamline with an Initial Starting Angle, θ = .05 rad

Fig. 6-4 - Laminar Heat Transfer Distribution Along a Streamline with an Initial Starting Angle, $\theta = .05$ rad

Section 7

CONCLUSIONS AND RECOMMENDATIONS

The Streamline Divergence program provides a means to make: (1) comparative evaluations of plume impingement due to different propulsion system configurations; (2) parametric studies; and (3) vehicle design studies. The Streamline Divergence program has demonstrated the capability to trace inviscid surface streamlines and calculate "outflow corrected" convective heating rates (laminar and turbulent) on surfaces subjected to exhaust plume impingement (or any other supersonic flow field). F.R. DeJarnette's streamline divergence methods are applied to determine the streamline patterns and calculate the corresponding "equivalent radii" or metric coefficients along each streamline for the particular geometry subjected to exhaust plume impingement. For purposes of demonstration the streamline divergence method has been applied to that of exhaust plume impingement on a cylinder. The incorporation of other geometries into the computer code is a straightforward procedure.

Convective heating rates are obtained from the boundary layer analysis for both the laminar and turbulent flow assumptions along each streamline. Eckert's reference enthalpy method is used to account for the temperature dependent property variations through the boundary layer. The local Reynolds number is output in order for the program user to choose the most applicable of the two heating rates.

Eckert's reference enthalpy menthod has been chosen for use in the convective heating rate analysis for demonstration purposes. Other methods do exist that may be applied to the problem of exhaust plume impingement and, can be incorporated into the computer code with a minimum of programming difficulty.

7-1

Section 8

REFERENCES

1-1 Penny, Morris M., and Carl J. Wojciechowski, "User's Manual and Description of a Computer Program for Calculating Heating Rates, Forces and Moments Acting on Bodies Immersed in Rocket Exhaust Plumes," LMSC-HREC D162867-II, Lockheed Missiles & Space Company, Huntsville, Ala., March 1971.

1-2 McAnally, J. V., and S. D. Smith, "Study of High Altitude Plume Impingement," LMSC-HREC TR D306389, Lockheed Missiles & Space Company, Huntsville, Ala., January 1973.

2-1 Svehla, Roger A., and Bonnie J. McBride, "FORTRAN IV Computer Program for Calculation of Thermodynamic and Transport Properties of Complex Chemical Systems," NASA TN D-7056, NASA-Lewis Research Center, Cleveland, Ohio.

2-2 Prozan, R. J., "Development of a Method-of-Characteristics Solution for the Supersonic Flow of an Ideal, Frozen, or Equilibrium Reacting Gas Mixture," LMSC-HREC D162220-III, Lockheed Missiles & Space Company, Huntsville, Ala., May 1970.

2-3 Smith, S. D., and A. W. Ratliff, "User's Manual — Variable O/F Method-of-Characteristics Program for Nozzles and Plume Flow Analysis," LMSC-HREC D162220-I, Lockheed Missiles & Space Company, Huntsville, Ala., May 1970.

2-4 Wojciechowski, C. J., and M. M. Penny, "Development of a High Altitude Plume Impingement Analysis for Calculating Heating Rates, Forces and Moments Acting on Bodies Immersed in Rocket Exhaust Plumes," LMSC-HREC D162867-1, Lockheed Missiles & Space Company, Huntsville, Ala., March 1971.

3-1 DeJarnette, F. R., "Calculation of Inviscid Surface Streamlines and Heat Transfer on Shuttle-Type Configurations," NASA CR 11921, Langley Research Center, Hampton, Va., August 1971.

3-2 Ahlberg, J. H., E. N. Nilson, and J. L. Walsh, The Theory of Splines and Their Applications, Academic Press, 1967.

3-3 Kays, W. M., Convective Heat and Mass Transfer, Chap. 14, McGraw Hill, New York, 1966.

3-4 Eckert, E. R. G., "Engineering Relations for Friction and Heat Transfer to Surfaces in High Velocity Flow," J. Aerospace Sci., August 1955, pp. 585-587.